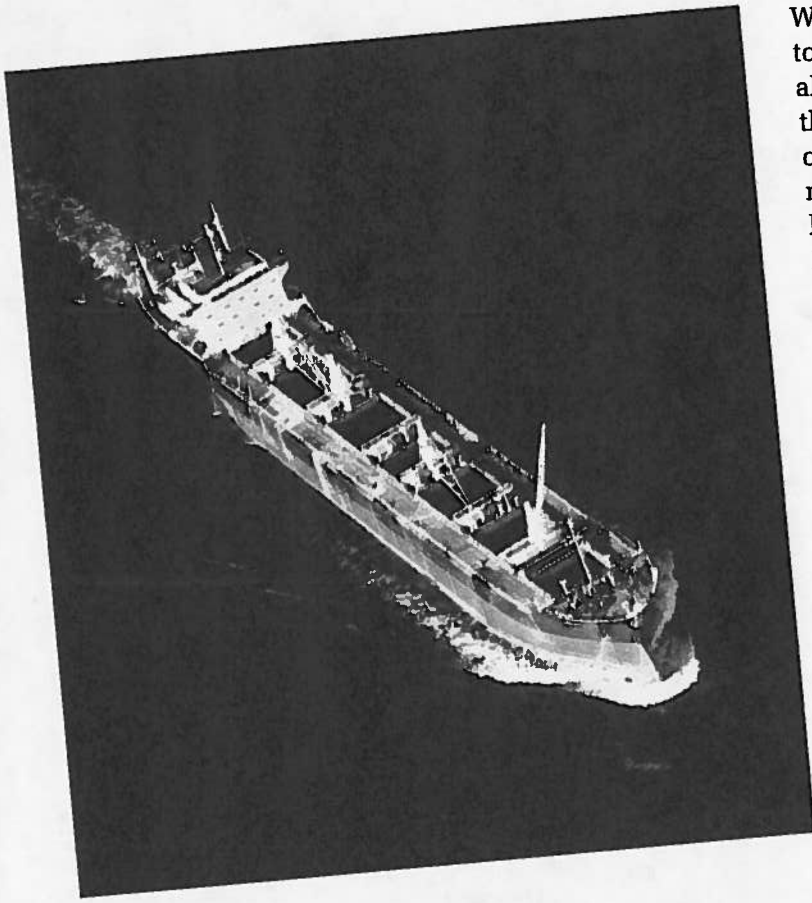
# 7.4 The Sine and Cosine Ratios



We depend on ships and aircraft to transport goods and people all over the world. If you were the captain of a ship or the pilot of an airplane, how could you make sure that you did not get lost in the middle of the ocean? In ancient times, this was a significant problem.

Today, navigational equipment such as Global Positioning System (GPS) devices makes it much easier to find your way around the planet. Even so, factors such as wind and water currents can sometimes complicate travel plans. How can trigonometry help when this happens?

## Investigate

**Tools**

- computer with *The Geometer's Sketchpad®*

OR

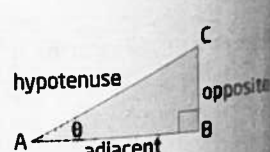
- TI-83 Plus or TI-84 Plus graphing calculator

OR

- grid paper
- protractor
- ruler

### What are the sine and cosine ratios?

In the last section, you learned the tangent ratio. In this activity, you will investigate two other important ratios. In addition to the opposite and adjacent sides, these ratios involve the third side of a right triangle: the hypotenuse.



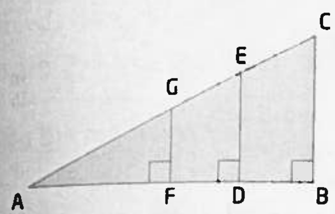

**1. a)** Draw a large right triangle $\triangle ABC$.

**b)** Measure the length of the side opposite $\angle A$.

**c)** Measure the length of the hypotenuse.

**d)** Calculate the ratio $\dfrac{\text{opposite}}{\text{hypotenuse}}$.

2. a) Create overlapping triangles by adding line segments parallel to one of the legs as shown.




b) Explain why these triangles are similar to the first one.

c) Measure and calculate the following for each similar triangle:
- length of the opposite side
- length of the hypotenuse
- the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$

d) Compare the $\frac{\text{opposite}}{\text{hypotenuse}}$ ratios for each triangle and describe what you notice.

3. a) Measure the length of the sides adjacent to $\angle A$ for each triangle.

b) Calculate the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ for each triangle. Describe what you notice.

4. a) Draw a new set of similar triangles, with a different value for $\angle A$.

b) Calculate and compare the ratio of $\frac{\text{opposite}}{\text{hypotenuse}}$ for each similar triangle.

c) Calculate and compare the ratio of $\frac{\text{adjacent}}{\text{hypotenuse}}$ for each similar triangle.

d) Repeat for another set of similar triangles.

The two ratios you have just explored are called the sine and cosine ratios. They are defined as

$$\text{sine } A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{and} \qquad \text{cosine } A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Together with the tangent ratio, they are the three **primary trigonometric ratios**.

5. **Reflect** Summarize what you have discovered about the three primary trigonometric ratios, using words and diagrams.

**Technology Tip**

To do step 2 using dynamic geometry software, follow these steps:
- Select the horizontal line segment. Construct a point on it.
- Select this segment and the new point, and construct a perpendicular line.
- Select the perpendicular line and the hypotenuse. Construct the point of intersection.
- Hide the perpendicular line. Construct a line segment connecting the two new points.

**Literacy Connections**

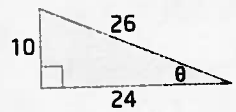
The short forms for sine and cosine are sin and cos.

**primary trigonometric ratios**
- sine, cosine, and tangent
- often abbreviated as sin, cos, and tan

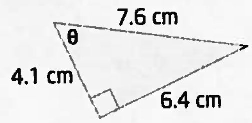
## Example 1 Find the Primary Trigonometric Ratios

Find the three primary trigonometric ratios for θ. Express the ratios as decimals, rounded to four decimal places.

**a)**

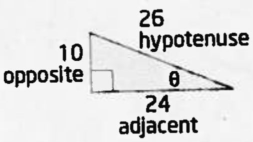



**b)**




### Solution

I need to identify the opposite, adjacent, and hypotenuse sides relative to the angle θ.



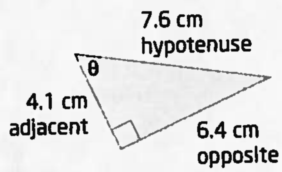

**a)**

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{10}{26} = \frac{5}{13} \doteq 0.3846$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{24}{26} = \frac{12}{13} \doteq 0.9231$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{24} = \frac{5}{12} \doteq 0.4167$$

**b)**




$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6.4}{7.6} \doteq 0.8421$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4.1}{7.6} \doteq 0.5395$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{6.4}{4.1} \doteq 1.5610$$

Since sine, cosine, and tangent are ratios, they have no units.

### Literacy Connections

A memory device (or mnemonic) for the three primary trigonometric ratios uses these short forms:

$S = \frac{O}{H}$ $C = \frac{A}{H}$ $T = \frac{O}{A}$

These short forms produce the nonsense phrase soh cah toa. This phrase may help you remember the formulas for the trigonometric ratios.

### Making Connections

Trigonometric ratios are often expressed with three or four digits of accuracy. This is to ensure that angles are found with enough precision. Consider the difference in the following two calculations:

$\tan^{-1}(0.2) \doteq 11.310°$ $\tan^{-1}(0.3) \doteq 16.699°$

Compare the difference between these two angles:

$\tan^{-1}(0.2492) \doteq 13.993°$ $\tan^{-1}(0.2493) \doteq 13.998°$

In both cases, the tangents of the angles differ by one decimal place. But in the second case, the angles are much closer together. Possible discrepancies due to rounding are reduced by carrying more digits until the final step in a calculation.

Just as with the tangent ratio, you can find the sine and cosine of an angle using a scientific or graphing calculator.

## Example 2 Find the Sine and Cosine of an Angle

Evaluate the following, to four decimal places.

**a)** sin 26° **b)** cos 75°

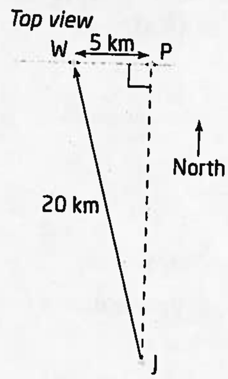
### Solution

Make sure that your calculator is in degree mode.

**a)** sin 26° ≐ 0.4384 **b)** cos 75° ≐ 0.2588

## Example 3 Find an Angle Using the Sine and Cosine Ratios

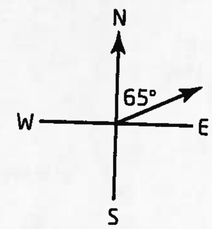
**a)** Captain Jack is navigating his ship to Port Harbour, which is directly north of the ship's location. To compensate for an easterly current, he aims for a point on shore that is 5 km west of Port Harbour. Assuming that the point on shore is 20 km from his position now, at what bearing must Jack head his ship?




**b)** Captain Jack is in communication with a submarine that is cruising at a depth of 400 m below sea level. If Jack's radar tells him that the submarine is 500 m from Jack, due north of his ship, at what angle is the submarine located with respect to Captain Jack's ship, to the nearest degree?

### Connections

*Bearing* is a navigational term that describes a direction. It is expressed a an angle in terms of north, south, east, and/or west.




This bearing can be described as N65°E, which is read as "65 degrees east of north."

### Solution

**a)** Captain Jack's ship, Port Harbour, and the western target form a right triangle.


For the unknown bearing angle, the opposite and hypotenuse sides are known.

Apply the sine ratio to find Jack's bearing.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= —$$

$$= 0.25$$

To find θ, calculate the inverse sine of 0.25.

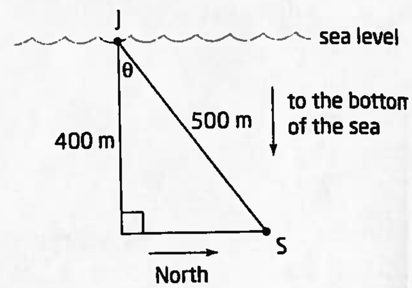
$\theta = \sin^{-1}(0.25)$

$\doteq 14.477°$

Jack must head his ship on a bearing of approximately N15°W.

**b)** Draw a diagram that shows the relative positions of Captain Jack's ship and the submarine. Captain Jack's ship, the submarine, and a line segment that points straight down from Captain Jack's ship form a right triangle.

*Side view, facing west*




For the unknown angle relating the submarine's position to Captain Jack, the adjacent and hypotenuse sides are known. Apply the cosine ratio to find this angle.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{400}{500}$$

$$\theta = \cos^{-1}\left(\frac{400}{500}\right)$$

$$\doteq 36.870$$

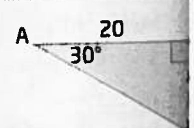
2nd [COS⁻¹] ( 400 + 500 ) =

or ( 400 + 500 ) 2nd [COS⁻¹]

The submarine is approximately 37° north of Captain Jack's ship wit respect to a line that points straight down to the bottom of the sea.

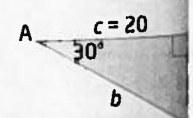
## Example 4 Solve a Right Triangle

Solve △ABC. Round side lengths to the nearest unit and angles to the nearest degree.




### Solution

Label the sides according to their corresponding angles.




Use the two known angles to find ∠C.

$$\angle A + \angle B + \angle C = 180°$$
$$30° + 90° + \angle C = 180°$$
$$120° + \angle C = 180°$$
$$\angle C = 180° - 120°$$
$$\angle C = 60°$$

Note that in a right triangle, the two acute angles are complementary. You can find ∠C by subtracting: 90° − 30° = 60°.

**Literacy Connections**

A triangle has six measures: three side lengths and three angle measures. To solve a triangle means to find all six of these values.

**Literacy Connections**

When naming parts of triangles, use capital letters to represent the angles and vertices and corresponding lowercase letters to represent opposite sides.

Use the cosine ratio to find side $b$.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{c}{b}$$

$$\cos 30^\circ = \frac{20}{b}$$

$b(\cos 30^\circ) = 20$ Multiply both sides by $b$.

$b = \dfrac{20}{\cos 30^\circ}$ Divide both sides by cos 30°.

$b \doteq 23.094$ 20 [÷] [COS] 30 [=] or 20 [÷] 30 [COS] [=]

Use the tangent ratio to find $a$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

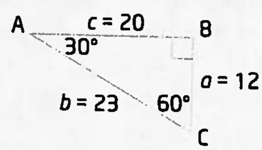
$$\tan A = \frac{a}{c}$$

$$\tan 30^\circ = \frac{a}{20}$$

$20(\tan 30^\circ) = a$ Multiply both sides by 20.

$a \doteq 11.547$ 20 [×] [TAN] 30 [=] or 20 [×] 30 [TAN] [=]

The diagram shows the solved triangle.




There is often more than one way to solve a right triangle. If you know the value of any side plus an additional side or angle, you can find the other measures. Notice, however, that some answers may be slightly different, due to rounding in the intermediate steps of a solution.

## Key Concepts

- The three primary trigonometric ratios are sine, cosine, and tangent. They are defined as follows:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- You can find any side length or angle measure of a right triangle if you know two pieces of information in addition to the right angle.